EXERCISE 5.3

In Problems 1–8, find exact solutions over the indicated intervals (x real and θ in degrees).

1. $2\cos x + 1 = 0, \quad 0 \le x < 2\pi$
2. $2\sin x + 1 = 0, \quad 0 \le x < 2\pi$
3. $2\cos x + 1 = 0, \quad$ all real x
4. $2\sin x + 1 = 0, \quad$ all real x
5. $\sqrt{2}\sin\theta - 1 = 0, \quad 0° \le \theta < 360°$
6. $2\cos\theta - \sqrt{3} = 0, \quad 0° \le \theta < 360°$
7. $\sqrt{2}\sin\theta - 1 = 0, \quad$ all θ
8. $2\cos\theta - \sqrt{3} = 0, \quad$ all θ

In Problems 9–14, solve each to four decimal places (x real and θ in degrees).

9. $4\tan\theta + 15 = 0, \quad 0° \le \theta < 180°$
10. $2\tan\theta - 7 = 0, \quad 0° \le \theta < 180°$
11. $5\cos x - 2 = 0, \quad 0 \le x < 2\pi$
12. $7\cos x - 3 = 0, \quad 0 \le x < 2\pi$
13. $5.0118\sin x - 3.1105 = 0, \quad$ all real x
14. $1.3224\sin x + 0.4732 = 0, \quad$ all real x

B *For Problems 15–26, find exact solutions (x real and θ in degrees).*

15. $\cos x = \cot x, \quad 0 \le x < 2\pi$
16. $\tan x = -2\sin x, \quad 0 \le x < 2\pi$
17. $\cos^2\theta = \frac{1}{2}\sin 2\theta, \quad$ all θ
18. $2\sin^2\theta + \sin 2\theta = 0, \quad$ all θ
19. $\tan(x/2) - 1 = 0, \quad 0 \le x < 2\pi$
20. $\sec(x/2) + 2 = 0, \quad 0 \le x < 2\pi$
21. $\sin^2\theta + 2\cos\theta = -2, \quad 0° \le \theta < 360°$
22. $2\cos^2\theta + 3\sin\theta = 0, \quad 0° \le \theta < 360°$
23. $\cos 2\theta + \sin^2\theta = 0, \quad 0° \le \theta < 360°$
24. $\cos 2\theta + \cos\theta = 0, \quad 0° \le \theta < 360°$
25. $4\cos^2 2x - 4\cos 2x + 1 = 0, \quad 0 \le x \le 2\pi$
26. $2\sin^2(x/2) - 3\sin(x/2) + 1 = 0, \quad 0 \le x \le 2\pi$

Solve Problems 27–30 (x real and θ in degrees). Compute inverse functions to four significant digits.

27. $4\cos^2\theta = 7\cos\theta + 2, \quad 0° \le \theta \le 180°$
28. $6\sin^2\theta + 5\sin\theta = 6, \quad 0° \le \theta \le 90°$
29. $\cos 2x + 10\cos x = 5, \quad 0 \le x < 2\pi$
30. $2\sin x = \cos 2x, \quad 0 \le x < 2\pi$

Solve Problems 31 and 32 for all real solutions. Compute inverse functions to four significant digits.

31. $\cos^2 x = 3 - 5\cos x$
32. $2\sin^2 x = 1 - 2\sin x$
33. Explain the difference between evaluating the expression $\cos^{-1}(-0.7334)$ and solving the equation $\cos x = -0.7334$.
34. Explain the difference between evaluating the expression $\tan^{-1}(-5.377)$ and solving the equation $\tan x = -5.377$.

C *Find exact solutions to Problems 35–38. [Hint: Square both sides at an appropriate point, solve, then eliminate any extraneous solutions at the end.]*

35. $\sin x + \cos x = 1, \quad 0 \le x < 2\pi$
36. $\cos x - \sin x = 1, \quad 0 \le x < 2\pi$
37. $\sec x + \tan x = 1, \quad 0 \le x < 2\pi$
38. $\tan x - \sec x = 1, \quad 0 \le x < 2\pi$

Exercise 5.3

1. $2\pi/3, 4\pi/3$
3. $2\pi/3 + 2k\pi, 4\pi/3 + 2k\pi$, k any integer
5. 45°, 135°
7. $45° + k(360°), 135° + k(360°)$, k any integer
9. 104.9314° **11.** 1.1593, 5.1239
13. $0.6696 + 2k\pi, 2.4720 + 2k\pi$, k any integer
15. $\pi/2, 3\pi/2$
17. $90° + k(180°), 45° + k(180°)$, k any integer
19. $\pi/2$ **21.** 180° **23.** 90°, 270°
25. $\pi/6, 5\pi/6, 7\pi/6, 11\pi/6$ **27.** 104.5°
29. 0.9987, 5.284
31. $0.9987 + 2k\pi, -0.9987 + 2k\pi$, k any integer
33. $\cos^{-1}(-0.7334)$ has exactly one value, 2.3941; the equation $\cos x = -0.7334$ has infinitely many solutions, which are found by adding $2\pi k$, k any integer, to each solution in one period of $\cos x$.
35. $0, \pi/2$ **37.** 0 **39.** 0.002613 sec **41.** 33.21°
43. 64.1° **45.** $(r, \theta) = (1, 30°), (1, 150°)$
47. $\theta = 45°$

[2] $2\sin x + 1 = 0$, $0 \le x \le 2\pi$

$\sin x = -\frac{1}{2}$

$\therefore\ x = \frac{7\pi}{6}$ or $x = \frac{11\pi}{12}$

NOTE that $-\frac{\pi}{6}$ is not an option since x required to be in $0 \le x \le 2\pi$

[4] $2\sin x + 1 = 0$, $x \in \mathbb{R}$

$\therefore\ x = -\frac{\pi}{6} + 2n\pi$, $x = \frac{7\pi}{6} + 2n\pi$, $n \in \mathbb{Z}$

[6] $2\cos\theta - \sqrt{3} = 0$, $0° \le \theta \le 360°$

$\cos\theta = \frac{\sqrt{3}}{2}$

$\therefore\ \theta = 30°$, $\theta = 330°$

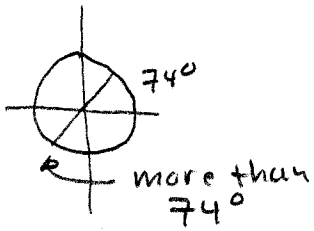
[8] $2\cos\theta - \sqrt{3} = 0$, $\theta \in \mathbb{R}$

$\therefore\ \theta = 30° + 360n$, $\theta = 330° + 360n$

[10] $2\tan\theta - 7 = 0$, $0° \le \theta \le 180°$

$\tan\theta = \frac{7}{2}$

$\theta = \tan^{-1}\left(\frac{7}{2}\right) = 74.0546°$

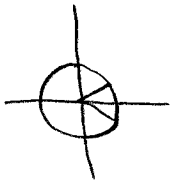


[12] $7\cos x - 3 = 0$, $0 \le x \le 2\pi$

$\cos x = \frac{3}{7}$

$x = \cos^{-1}\left(\frac{3}{7}\right)$

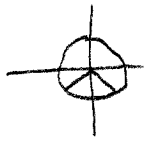
$x = 1.1279$, 5.1553



$2\pi - 1.1279 =$

[14] $1.3224 \sin x + 0.4732 = 0$, $x \in \mathbb{R}$

$$\sin x = \frac{-0.4732}{1.3224}$$



$\therefore$ $x = -0.3659 + 2n\pi$ or $x = 3.5075 + 2\pi$

[16] $\cos x = \cot x$, $0 \leq x \leq 2\pi$

$$\cos x = \frac{\cos x}{\sin x}$$

$$\sin x \cos x = \cos x$$

$$\cos x(\sin x - 1) = 0$$

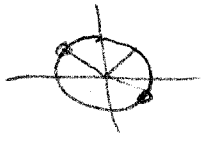
$\cos x = 0$ or $\sin x = 1$

$\therefore$ $x = \frac{\pi}{2}, \frac{3\pi}{2}$

[18] $2\sin^2\theta + \sin 2\theta = 0$, all θ

$\equiv 2\sin^2\theta + 2\sin\theta\cos\theta = 0$

$\equiv 2\sin\theta(\sin\theta + \cos\theta) = 0$



$\Rightarrow$ $2\sin\theta = 0$ or $\sin\theta + \cos\theta = 0$

$\Rightarrow$ $\theta = 360^\circ n$ or $\sin\theta = -\cos\theta$

$\theta = 135, \theta = 315$

$\therefore$ $\theta = 135^\circ + 360n$ or $\theta = 315^\circ + 360n$ or $\theta = 360^\circ n$

[20] $\sec\left(\frac{x}{2}\right) + 2 = 0$, $0 \le x \le 2\pi$

$\sec\left(\frac{x}{2}\right) = -2$

$\sec\left(\frac{2\pi}{3} + 2n\pi\right) = \sec\left(\frac{x}{2}\right)$ or $\sec\left(\frac{4\pi}{3} + 2n\pi\right) = \sec\left(\frac{x}{2}\right)$

$\therefore \quad x = \frac{4\pi}{3} + 4n\pi$, $x = \frac{8\pi}{3} + 4n\pi$

[22] $2\cos^2\theta + 3\sin\theta = 0$, $0° \le \theta \le 360°$

$2[1 - \sin^2\theta] + 3\sin\theta = 0$

$2 - 2\sin^2\theta + 3\sin\theta = 0$

$2\sin^2\theta - 3\sin\theta - 2 = 0$

$(2\sin\theta + 1)(\sin\theta - 2) = 0$

$2\sin\theta + 1 = 0$ OR $\sin\theta - 2 = 0$

$\sin\theta = -\frac{1}{2}$ $\quad \phi$

$\theta = -30 + 360n = 330° + 360n$

OR

$\theta = 180 + 30 + 360n = 210° + 360n$

$\therefore \quad \theta = 210°$, $\theta = 330°$

[24] $\cos 2\theta + \cos\theta = 0$, $0° \le \theta \le 360°$

$2\cos^2\theta + \cos\theta - 1 = 0$

$(2\cos\theta - 1)(\cos\theta + 1) = 0$

$2\cos\theta - 1 = 0$ or $\cos\theta = -1$

$\cos\theta = \frac{1}{2}$ | $\theta = 180°$

$\theta = 60°$ or $300°$

$\therefore \quad \theta = 60°, 180°, 300°$

[26] $2\sin^2\left(\frac{x}{2}\right) - 3\sin\left(\frac{x}{2}\right) + 1 = 0$, $0 \le x \le 2\pi$

$\left(2\sin\left(\frac{x}{2}\right) - 1\right)\left(\sin\left(\frac{x}{2}\right) - 1\right) = 0$

$2\sin\left(\frac{x}{2}\right) - 1 = 0$ or $\sin\left(\frac{x}{2}\right) - 1 = 0$

$\sin\left(\frac{x}{2}\right) = \frac{1}{2}$ | $\sin\left(\frac{x}{2}\right) = 1$

$\frac{x}{2} = \frac{\pi}{6}$ or $\frac{x}{2} = \frac{5\pi}{6}$ | $\frac{x}{2} = \pi/2$

$x = \frac{\pi}{3}$ or $x = \frac{5\pi}{3}$ | $x = \pi$

$\therefore \quad x = \frac{\pi}{3}, \pi, \frac{5\pi}{3}$

[28]

$6\sin^2\theta + 5\sin\theta - 6 = 0 \qquad 0° \le \theta \le 90°$

$$\sin\theta = \frac{-5 \pm \sqrt{25+4\cdot6\cdot6}}{12}$$

$$= \frac{-5 \pm \sqrt{169}}{12}$$

$$= \frac{-5 \pm 13}{12}$$

$$= \frac{8}{12} = \frac{2}{3} \quad \text{OR} \quad \frac{-18}{12} > 1$$

$\emptyset$

$$\sin\theta = \frac{2}{3}$$

$$\theta = 41.8087°$$

or $\theta = 180 - 41.8087 = 138.1913$

$\therefore \quad \theta = 41.8087° \text{ or } \theta = 138.1913°$

[30] $2\sin x = \cos 2x, \qquad 0 \le x \le 2\pi$

$2\sin x - \cos 2x = 0$

$2\sin x - (1 - 2\sin^2 x) = 0$

$2\sin x - 1 + 2\sin^2 x = 0$

$2\sin^2 x + 2\sin x - 1 = 0$

$$\sin x = \frac{-2 \pm \sqrt{4+8}}{4} = \frac{-2 \pm 2\sqrt{3}}{4} = \frac{-1+\sqrt{3}}{2}, \frac{-1-\sqrt{3}}{2}$$

So $\sin x = \frac{-1+\sqrt{3}}{2}$ or $\sin x = \frac{-1-\sqrt{3}}{2}$

$\Rightarrow$ $\boxed{x = 0.3747}$ or $\emptyset$

[36] $\sin x + \cos x = 1$, $0 \le x \le 2\pi$

$\sin^2 x + 2\sin x \cos x + \cos^2 x = 1$

$2 \sin x \cos x = 0$

$\sin 2x = 0$

$2x = 0$ or $2x = 2\pi$

$x = 0$, $x = \pi$

$\sin 0 + \cos 0 = 1$ ✓
$\sin \pi + \cos \pi = -1$ NO

$\therefore \quad x = 0$

[38] $\tan x - \sec x = 1$ $0 \le x \le 2\pi$

$\tan^2 x - 2\tan x \sec x + \sec^2 x = 1$

$\tan^2 x - 2\tan x \sec x + \sec^2 x - 1 = 0$

$2\tan^2 x - 2\tan x \sec x = 0$

$\tan x (\tan x - \sec x) = 0$

$\tan x = 0$ OR $\tan x - \sec x = 0$

$\frac{\sin x}{\cos x} = 0$

$\sin x = 0$

$x = 0, \pi, 2\pi$

$\tan x = \sec x$

$\frac{\sin x}{\cos x} = \frac{1}{\cos x}$

$\sin x = 1$

$x = \frac{\pi}{2}$

$\tan 0 - \sec 0 = 0 - 1 \neq 1$
$\tan \pi/2 - \sec \pi/2 =$ UNDEF
$\tan \pi - \sec \pi = 0 - (-1) = 1$ ✓

$\therefore \quad x = \pi$